Paper Reference(s)

6677/01

Edexcel GCE

Mechanics M1

Advanced/Advanced Subsidiary

Wednesday 3 June 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M1), the paper reference (6677), your surname, other name and signature.

Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 8 questions in this question paper.

The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

1. Particle P of mass m and particle Q of mass km are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision the speed of P is $5u$ and the speed of Q is u. Immediately after the collision the speed of each particle is halved and the direction of motion of each particle is reversed.

Find

(a) the value of k,

(3)

(b) the magnitude of the impulse exerted on P by Q in the collision.

(3)

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2. A small stone is projected vertically upwards from a point O with a speed of 19.6 m s^{-1}. Modelling the stone as a particle moving freely under gravity,

(a) find the greatest height above O reached by the stone,

(2)

(b) find the length of time for which the stone is more than 14.7 m above O.
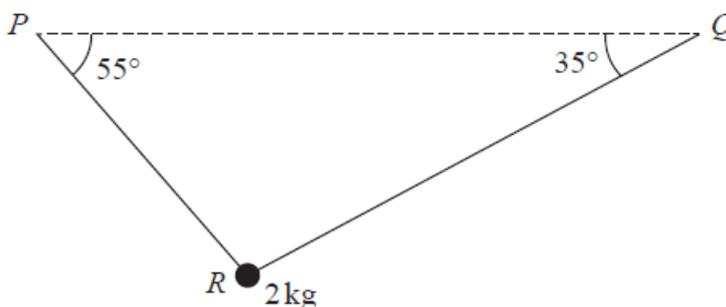
(5)

3.

Figure 1

A particle of mass 2 kg is suspended from a horizontal ceiling by two light inextensible strings, PR and QR. The particle hangs at R in equilibrium, with the strings in a vertical plane. The string PR is inclined at 55° to the horizontal and the string QR is inclined at 35° to the horizontal, as shown in Figure 1.

Find

(i) the tension in the string PR,

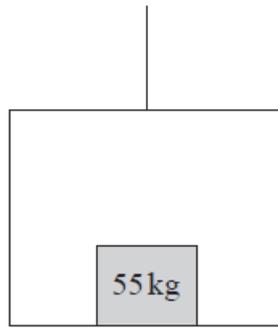(ii) the tension in the string QR.

(7)

4.

Figure 2

A lift of mass 200 kg is being lowered into a mineshaft by a vertical cable attached to the top of the lift. A crate of mass 55 kg is on the floor inside the lift, as shown in Figure 2. The lift descends vertically with constant acceleration. There is a constant upwards resistance of magnitude 150 N on the lift. The crate experiences a constant normal reaction of magnitude 473 N from the floor of the lift.

(*a*) Find the acceleration of the lift.

(3)

(*b*) Find the magnitude of the force exerted on the lift by the cable.
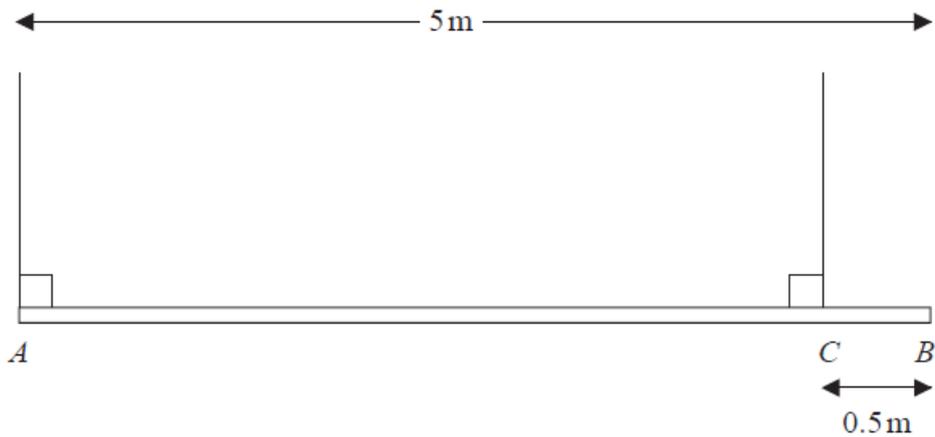
(4)

5.

Figure 3

A beam AB has length 5 m and mass 25 kg. The beam is suspended in equilibrium in a horizontal position by two vertical ropes. One rope is attached to the beam at A and the other rope is attached to the point C on the beam where $CB = 0.5$ m, as shown in Figure 3. A particle P of mass 60 kg is attached to the beam at B and the beam remains in equilibrium in a horizontal position. The beam is modelled as a uniform rod and the ropes are modelled as light strings.

(*a*) Find

    (i)   the tension in the rope attached to the beam at A,

    (ii) the tension in the rope attached to the beam at C.

(6)

Particle P is removed and replaced by a particle Q of mass M kg at B. Given that the beam remains in equilibrium in a horizontal position,

(*b*) find

    (i)   the greatest possible value of M,

    (ii) the greatest possible tension in the rope attached to the beam at C.

(6)

6. A particle P is moving with constant velocity. The position vector of P at time t seconds $(t \geq 0)$ is $\mathbf{r}$ metres, relative to a fixed origin O, and is given by

$$\mathbf{r} = (2t - 3)\mathbf{i} + (4 - 5t)\mathbf{j}.$$

(*a*) Find the initial position vector of P.

(1)

The particle P passes through the point with position vector $(3.4\mathbf{i} - 12\mathbf{j})$ m at time T seconds.

(*b*) Find the value of T.

(3)

(*c*) Find the speed of P.

(4)

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7. A train travels along a straight horizontal track between two stations, A and B. The train starts from rest at A and moves with constant acceleration 0.5 m s^{-2} until it reaches a speed of V m s^{-1}, $(V < 50)$. The train then travels at this constant speed before it moves with constant deceleration 0.25 m s^{-2} until it comes to rest at B.

(*a*) Sketch a speed-time graph for the motion of the train between the two stations A and B.

(2)

The total time for the journey from A to B is 5 minutes.

(*b*) Find, in terms of V, the length of time, in seconds, for which the train is

   (i)   accelerating,

   (ii)   decelerating,

   (iii) moving with constant speed.

(5)

Given that the distance between the two stations A and B is 6.3 km,

(*c*) find the value of V.

(6)

8.
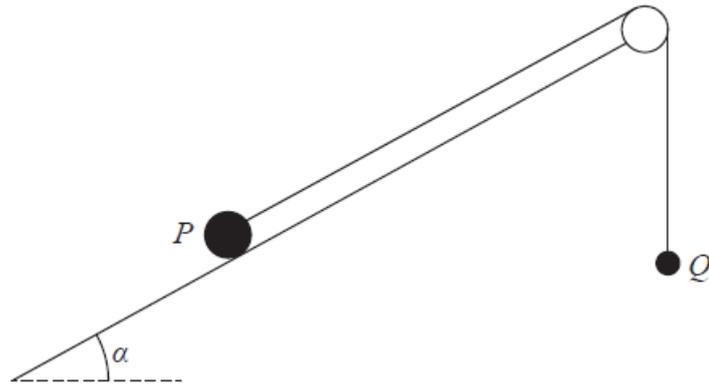
Figure 4

Two particles P and Q have mass 4 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough plane, which is inclined to the horizontal at an angle α where $\tan \alpha = \dfrac{4}{3}$. The coefficient of friction between P and the plane is 0.5. The string lies along the plane and passes over a small smooth light pulley which is fixed at the top of the plane. Particle Q hangs freely at rest vertically below the pulley. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 4. Particle P is released from rest with the string taut and slides down the plane.

Given that Q has not hit the pulley, find

(*a*) the tension in the string during the motion,

(11)

(*b*) the magnitude of the resultant force exerted by the string on the pulley.

(4)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1(a)	$$m.5u - kmu = -\frac{m.5u}{2} + \frac{km.u}{2}$$ $$k = 5$$	M1 A1 A1 (3)
(b)	For $P: I = m\left(\frac{5u}{2} - -5u\right)$ OR For $Q: I = km\left(\frac{u}{2} - -u\right)$ $\qquad = \frac{15mu}{2} \qquad\qquad\qquad\qquad = \frac{15mu}{2}$	M1 A1 A1 (3) **6**
	Notes	
1(a)	M1 for attempt at CLM equation, with correct no. of terms, dimensionally correct. Allow consistent extra g's and cancelled *m*'s and *u*'s and sign errors. First A1 for a correct equation with or without *m*'s and *u*'s Second A1 for $k = 5$ **N.B.** They may find the impulse on each particle and then equate the impulses to produce an equation. Apply the scheme to this equation.	
1(b)	M1 for attempt at impulse = difference in momenta, for either particle, (must be considering *one* particle) (M0 if g's are included or if *m* or *u* omitted) Allow $\pm m(\frac{5}{2}u - 5u)$ or $\pm km(\frac{1}{2}u - u)$. First A1 for $\pm m(\frac{5}{2}u - -5u)$ or $\pm km(\frac{1}{2}u - -u)$ A1 for 7.5*mu* oe cao (-7.5*mu* is A0) Allow change of sign at end to obtain magnitude	

Question Number	Scheme	Marks
2(a)	$$0^2 = 19.6^2 - 2 \times gH$$ $$H = 19.6\text{m (20)}$$	M1 A1 (2)
(b)	$$14.7 = 19.6t - \tfrac{1}{2}gt^2$$ $$t^2 - 4t + 3 = 0$$ $$(t-1)(t-3) = 0$$ $$t = 1 \text{ or } 3; \quad \text{Answer 2 s}$$	M1 A1 DM1 A1; A1 (5)
		7
2(b) **ALT 1**	(their h - 14.7) $= \frac{1}{2} g\, t^2$ **OR** $v^2 = 19.6^2 - 2g \times 14.7 \Rightarrow v = (\pm)\ 9.8$ $t = 1$ and $0\ 9.8 - 9.8\, t \Rightarrow t = 1$ Total $= 2 \times$ their 1 $= 2$ s	M1 A1 A1 DM 1 A1
2(b) **ALT 2/3**	$v^2 = 19.6^2 - 2g \times 14.7$ $v = \pm\ 9.8$ **EITHER:** $-9.8 = 9.8 - gT$ $T = 2$ **OR:** $0 = 9.8t - \frac{1}{2}g\,t^2$ $t = (0)\ $ or 2	M1 A1 DM1 A1 A1 DM1 A1 A1
	Notes	
2(a)	M1 is for a complete method (which could involve use of two *suvat* equations) for finding H i.e. for an equation in H *only*, condone sign errors A1 for 19.6 or 20 <u>correctly obtained</u> ($2g$ is A0)	
2(b)	First M1 is for a quadratic equation in t only (where t is time at 14.7 above *O*) First A1 for a correct equation Second DM1, dependent on first M1, for solving for t Second A1 for <u>both</u> values of t, 1 and 3. **N.B**. If answer(s) are wrong or have come from an incorrect quadratic, and the quadratic formula has been used, M1 can only be awarded if there is clear evidence that the correct formula has been used. If their expression is not correct for their quadratic, allow a slip but only if <u>we see an attempt to substitute into a stated correct formula.</u> Third A1 for 2 s **N.B.** Obtaining $t = 1$ at $s = 14.7$ (above *O*) only, can score max M1 A1	

Question Number	Scheme	Marks
3	$$T_P \cos 55 = T_Q \cos 35$$ $$T_P \sin 55 + T_Q \sin 35 = 2g$$ Eliminating T_P or T_Q $$T_P = 16\text{N or } 16.1\text{N}; \quad T_Q = 11\text{N or } 11.2\text{N}$$	M1 A1 M1 A1 M1 A1 A1 **7**
ALT 1	$\left(\text{Along } RP\right) \ T_P = 2g \cos 35° = 16\text{N or } 16.1\text{N}$ $\left(\text{Along } RQ\right) \ T_Q = 2g \cos 55° = 11\text{N or } 11.2\text{N}$	M1 M1 A1 A1 M1 A1 A1
	Notes	
	First M1 for resolving horizontally with correct no. of terms and both T_P and T_Q terms resolved. (M0 if they assume $T_P = T_Q$) First A1 for a correct equation. Second M1 for resolving vertically with correct no. of terms and both T_P and T_Q terms resolved. (M0 if they assume $T_P = T_Q$) Second A1 for a correct equation. Third M1 (independent) for eliminating either T_P or T_Q <u>Third</u> A1 for $T_P = 16$ (N) or 16.1 (N) <u>Fourth</u> A1 for $T_Q = 11$ (N) or 11.2 (N) N.B. If both are given to more than 3SF, deduct the third A1.	
ALT 1	**Alternative 1 (resolving along each string)** First M2 for resolving along one of the strings (e.g. $T_P = 2g\cos 35°$) First A1 for a correct equation ($T_P = 2g\sin 35°$ scores M2A0A0) <u>Third</u> A1 for $T_P = 16$ (N) or 16.1 (N) Third M1 for resolving along the other string (e.g. $T_Q = 2g\cos 55°$) Second A1 for a correct equation ($T_Q = 2g\sin 55°$ scores M1A0A0) <u>Fourth</u> A1 for $T_Q = 11$ (N) or 11.2 (N)	
ALT 2	**Alternative 2 (using a Triangle of Forces)** Both of the equations in Alternative 1 could come from using *sohcahtoa* or The Sine Rule on a triangle of forces, so mark in the same way. Note that, in either case, once they have found either T_P or T_Q , they could then use $T_P = T_Q \tan 55°$ or $T_Q = T_P \tan 55°$ to find the other one. <u>(Note that both of these are equivalent to the horizontal resolution) or Pythagoras.</u> e.g. $T_P = 2g\cos 35°$ — M2 First A1 $\qquad = 16$ (N) or 16.1 (N) — <u>Third</u> A1 $\quad T_Q = T_P \tan 35°$ or $\sqrt{(2g)^2 - (T_P)^2}$ — M1 Second A1 $\qquad = 11$ (N) or 11.2 (N) — <u>Fourth</u> A1	

N.B. If they are clearly using The Sine Rule but have say 35º, 55º and 80º in their triangle, all 3 M marks would be available and at most 1 A mark

e.g. $T_P = \dfrac{2g\sin 55}{\sin 80}$ M2 A0A0

$T_Q = \dfrac{T_P \sin 35}{\sin 55}$ M1 SecondA1 A0

Question Number	Scheme	Marks
4(a)	For crate, $55g - 473 = 55a$	M1 A1
	$a = 1.2\text{m s}^{-2}$	A1 (3)
(b)	For system, $55g + 200g \pm T - 150 = 255a$	M1 A2
	M agnitude $= 2040$ N or 2000 N	A1
	OR	
	For lift, $200g + 473 - 150 \pm T = 200a$	M1 A2
	M agnitude $= 2040$ N or 2000 N	A1 (4)
		7
	Notes	
4(a)	M1 for an equation in a only, with usual rules. First A1 for a correct equation Second A1 for 1.2 (m s^{-2}). Allow -1.2 (m s^{-2}) if appropriate	
4(b)	M1 for an equation, in T and a , for the system or the lift only, with usual rules. (a does not need to be a numerical value) A2 (-1 each error) for a correct equation (**Allow** $\pm T$). We do **not** need to see a numerical value for a. Third A1 for 2040 (N) or 2000 (N) **N.B.** In both parts of this question use the mass which is being used to guide you as to which part of the system is being considered.	

Question Number	Scheme	Marks
5(a)	$T_A + T_C = 85g$	M1 A1
	OR $M(A),\ 25g \times 2.5 + 60g \times 5 = 4.5 \times T_C$	
	OR $M(C),\ T_A \times 4.5 + 60g \times 0.5 = 25g \times 2$	
	OR $M(B),\ T_A \times 5 + T_C \times 0.5 = 25g \times 2.5$	M1 A1
	OR $M(G),\ T_A \times 2.5 + 60g \times 2.5 = 2 \times T_C$	
	$T_A = \dfrac{40g}{9} = 44\text{N or } 43.6\text{N};\ T_C = \dfrac{725g}{9} = 790\text{N or } 789 \text{ N}$	A1; A1 (6)
(b)	$M(C),\ 25g \times 2 = Mg \times 0.5$	M1 A1
(i)	$M = 100$	A1
(ii)	$T_c = 25g + 100g$	M1 A1
	$T_c = 125g\ (1200 \text{ or } 1230)\text{N}$	B1 (6) **12**
	Notes	
5(a)	First M1 for a moments or vertical resolution equation, with correct no. of terms and dimensionally correct.	
	First A1 for a correct equation.	
	Second M1 for a moments equation, with correct no. of terms and dimensionally correct.	
	Second A1 for a correct equation.	
	Third A1 for 44 (N) or 43.6 (N) or 40g/9	
	Fourth A1 for 790 (N) or 789 (N) or 725g/9	
	Deduct 1 mark for inexact multiples of *g*	
	N.B. If they assume that both tensions are the same, can only score max M1 in (a) for *M(A)* or *M(C)*.	
	If a vertical resolution is used, please give marks for this equation FIRST. If not, enter marks for each moments equation in the order in which they appear.	
5(b)	**SCHEME CHANGE**	
	B1 BECOMES THE FOURTH A1	
	First M1 for a moments equation with $T_A = 0$	
	First A1 for a correct equation	
	Second A1 for $M = 100$	
	Second M1 for a(nother) moments or vertical resolution equation with $T_A = 0$	
	Third A1 for a correct equation	
	Fourth A1 (B1) for $T_C = 125g$ or 1230 (N) or 1200 (N)	
	N.B. Some candidates may need to solve 2 simult. equations in M and T_C and so will earn the 'equation' marks before they earn Second and Fourth A (B) marks.	
	If a vertical resolution is used, please give marks for this equation SECOND. If not, enter marks for each moments equation in the order	

in which they appear.

The possible equations are:
$T_C = 25g + Mg$
M(C), $25g \times 2 = Mg \times 0.5$
M(A), $25g \times 2.5 + 5Mg = 4.5\ T_C$
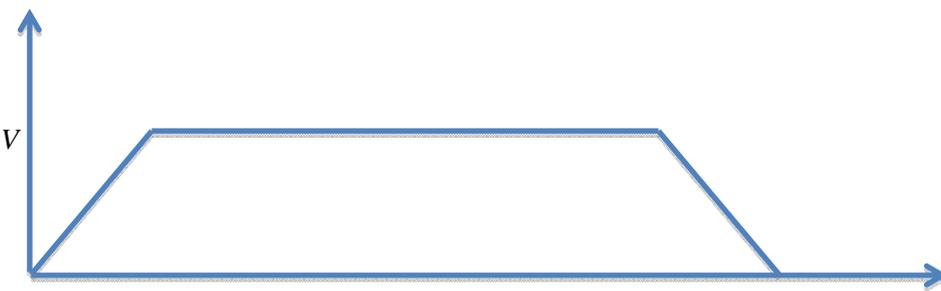M(B), $25g \times 2.5 = T_C \times 0.5$
M(G), $T_C \times 2 = Mg \times 2.5$

Any two of these can each earn M1A1 (M0 if incorrect no. of terms)
Then Second A1 for $M = 100$
And Fourth A1 (B1) for $T_C = 125g$ or 1230 or 1200

N.B. No marks in (b) if they use any answers from (a) or $M = 60$

Question Number	Scheme	Marks	
6(a)	$\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j})$ m	B1	(1)
(b)	$3.4 = 2T - 3$ or $-12 = 4 - 5T$	M1 A1	
	$T = 3.2$	A1	(3)
(c)	$\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j}) + t(2\mathbf{i} - 5\mathbf{j})$	M1	
	$\mathbf{v} = (2\mathbf{i} - 5\mathbf{j})$	A1	
	speed $= \sqrt{\left(2^2 + (-5)^2\right)} = \sqrt{29} = 5.4\text{m s}^{-1}$ or better	M1 A1	(4)
			8
Alt (c)	$|\mathbf{s}| = \sqrt{6.4^2 + (-16)^2} = 17.23...$	M1 A1	
	$\therefore$ speed $= \dfrac{17.23}{3.2} = 5.4$ or better	M1 A1	(4)
	Notes		
6(a)	**Allow column vectors throughout.** B1 for $(-3\mathbf{i} + 4\mathbf{j})$ (m)		
(b)	M1 for a clear attempt at either 3.4 (**i**)$= (2T - 3)$ (**i**) or -12(**j**) $= (4 - 5T)$ (**j**) First A1 for a correct equation (either) <u>without **i**'s and **j**'s</u> A1 for 3.2 oe **N.B.** $T = \dfrac{6.4\mathbf{i} - 16\mathbf{j}}{2\mathbf{i} - 5\mathbf{j}} = 3.2$ scores M1A1A1 <u>BUT</u> if RHS is not a single number, then M0. Also, if they get 3.2 and another value and don't clearly choose 3.2 then A0		
(c)	First M1 for a complete method for finding **v** e.g. $\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j}) + t(2\mathbf{i} - 5\mathbf{j})$ so $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j}$ OR: $\mathbf{v} = \dfrac{(3.4\mathbf{i} - 12\mathbf{j}) - (-3\mathbf{i} + 4\mathbf{j})}{\text{their } T}$ OR: $\mathbf{v} = \dfrac{d\mathbf{r}}{dt} = 2\mathbf{i} - 5\mathbf{j}$ First A1 for $2\mathbf{i} - 5\mathbf{j}$; M1A1 can be awarded for $2\mathbf{i} - 5\mathbf{j}$ <u>only.</u> Second M1 for attempt to find magnitude of their **v**, i.e. $\sqrt{2^2 + (-5)^2}$ Second A1 for $\sqrt{29}$ or 5.4 or better **OR** First M1 for attempt to find distance travelled: $d = \sqrt{(-3 - 3.4)^2 + (4 - -12)^2}$ First A1 if correct Second M1 for their d / their T Second A1 for $\sqrt{29}$ or 5.4 or better		

Question Number	Scheme	Marks
7(a)		B1 (shape) B1 (*V*) (2)
(b) **(i) (ii)**	$\dfrac{V}{t_1} = \dfrac{1}{2} \;\Rightarrow\; t_1 = 2V$ s; $t_2 = 4V$ s	M1 A1; A1
(iii)	$t_3 = 300 - 2V - 4V = 300 - 6V$ s	M1 A1 (5)
(c)	$6300 = \dfrac{V(300 + 300 - 6V)}{2}$ or $\dfrac{1}{2}2V.V + (300 - 6V).V + \dfrac{1}{2}4V.V$ $V^2 - 100V + 2100 = 0$ $(V - 30)(V - 70) = 0$ $V = 30$ or 70 $V = 30\;(<50)$	M1 **A1 ft** A1 M1 A1 A1 (6) **13**
Notes		
7(a)	B1 for a trapezium with line starting and finishing on the *t*-axis B1 for *V* correctly marked	
(b)	First M1 for a correct method First A1 for *V*/0.5 oe Second A1 for *V*/0.25 oe Second M1 for (300 – sum of previous answers) Allow 5 instead of 300. Third A1 for 300 – 6*V* oe	
(c)	First M1 for using the area under the curve (distance travelled) to form an equation in *V* only. (Allow use of 6.3 but must see ½ used at least once in their expression.) First A1 **ft on their answers in (b)** for a correct equation so must have used 6300 not 6.3 Second A1 for correct equation in form $aV^2 + bV + c = 0$ Second M1 for solving a 3 term quadratic. (Can be implied by correct answers) Second A1 for either 30 or 70	

	Third A1 for 30 as final answer. **N.B**. If answer(s) are wrong or have come from an incorrect quadratic, and the quadratic formula is used, M1 can only be awarded if there is clear evidence that the correct formula has been used. i.e. <u>we need to see numbers substituted into a stated correct formula.</u>	

Question Number	Scheme	Marks
8(a)	$$R = 4g\cos\alpha$$	M1 A1
	$$T - 0.5g = 0.5a$$	M1 A1
	$$4g\sin\alpha - T - F = 4a$$	M1 A1
	(OR: $4g\sin\alpha - F - 0.5g = 4.5a$)	
	$$F = \frac{1}{2}R; \quad \sin\alpha = \frac{4}{5} \quad \text{or} \quad \cos\alpha = \frac{3}{5}$$	B1; B1
	Eliminating a or finding a	M1
	Solving for T (must have had an a)	M1
	$$T = \frac{2g}{3}\,\text{N or 6.5N or 6.53N}$$	A1
		(11)
(b)	$$\text{Magnitude} = 2T\cos\left(\frac{90-\alpha}{2}\right)$$	M1 A1
	$$= 2 \times \frac{2g}{3} \times \frac{3}{\sqrt{10}}\ (0.94868..)$$	**A1 ft on T**
	$$= 12N \text{ or } 12.4N \quad \left(\frac{4g}{\sqrt{10}}\right)$$	A1 (4)
		15
	Notes	
8(a)	First M1 for resolving perp to plane, with usual criteria	

First A1 for a correct equation

Second M1 for resolving vertically, with usual criteria

Second A1 for a correct equation, in terms of a and T

Third M1 for resolving parallel to the slope, with usual criteria.

Third A1 for a correct equation , in terms of a, F and T

N.B. Their a could be UP the slope in which case all 4 marks for the 2 equations are available with $-a$ replacing a, provided they are consistent. If they are inconsistent, then assume the vertical resolution is the correct one and mark accordingly.

Either of the above two equations can be replaced by the 'whole system' equation

N.B. If they use a = 0, in any of the above 3 equations, and they use the equation to find T, they lose both marks for that equation, and they lose the two M marks for eliminating and solving.

First B1 for $F = \frac{1}{2}R$ seen or implied;

Second B1 for $\sin\alpha = 0.8$ or $\cos\alpha = 0.6$ seen or implied. Allow close approximations if $\alpha = 53.1°\ldots$ used.

Fourth M1 independent for eliminating a or finding a.

Fifth M1 for solving for T but must have had an a.

Fourth A1 for 2g/3, 6.5 or 6.53.

(b)

First M1 for a complete method for finding the magnitude of the resultant (**N.B.** M0 if same tensions used)

$2T \cos\left(\dfrac{90° - \alpha}{2}\right)$ .Allow sin/cos confusion and allow $2T \cos\left(\dfrac{\alpha}{2}\right)$

OR $\sqrt{(T + T\sin\alpha)^2 + (T\cos\alpha)^2}$. Allow sin/cos confusion and allow omission of √ sign, but only if $R^2 = $ ……. is included

OR $\sqrt{T^2 + T^2 - 2T^2 \cos(90° + \alpha)}$. Allow ($90° - \alpha$) but must be cos and and allow omission of √ sign, but only if $R^2 = $ ……is included

OR $\dfrac{T \sin(90 + \alpha)}{\sin\left(\dfrac{90° - \alpha}{2}\right)}$. (**Sine Rule**) Allow sign errors in angles but must

be sin

First A1 for correct expression in terms of T and α

Second A1, **ft** on their T, for a 'correct' **single numerical** answer

Third A1 cao for 12 (N) or 12.4 (N)